GROWTH LAW OF A SPHERICAL SECOND PHASE AS GOVERNED BY SIMULTANEOUS HEAT AND MULTI-COMPONENT MASS TRANSFER LIMITATIONS—II

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Abstract—In Part II, new theoretical treatments of the growth of a slowly moving spherical second phase as governed by simultaneous heat and multi-component mass transfer limitations are demonstrated. The new method is a direct extension of [1–3]. It demonstrates that the solution to these complex coupled cases can be related to the available uncoupled cases. Thus, treating the so-called "impurities" as components in the surrounding first phase, our results should include the growth of a slowly moving spherical second phase as governed by simultaneous heat and mass transfer limitations in the presence of impurities as asymptotic cases.

STATEMENT OF THE PROBLEM

THE PROBLEM under consideration in Part II is as follows: A spherical second phase of size, R_0, is produced in a N-component environment, i.e. the surrounding first phase, at time $t = 0$. The second phase can be a bubble (gas), a droplet (liquid), or a particle (solid). At time $t = 0$, the entire second phase is assumed to have attained a certain proper equilibrium temperature T_w, i.e. the wet bulb temperature, and remain at this temperature throughout the growth process. That is, one assumes that throughout the entire transient growth process a constant T_w exists, corresponding to a set of constant surface concentrations, $C_{\mathrm{sat}\,1}(T_w, C_{w2}, C_{w3}, \ldots, C_{wN-1})$ and C_{wi} $(i = 2, 3, \ldots, N-1)$, which must be found as part of the problem solution (see Discussion). At times $t > 0$, the spherical second phase starts to grow due to both heat and N-component mass transfer driving forces and move slowly in the surrounding first phase. The center of the second phase sphere is assumed to move at a velocity U_∞ relative to stationary coordinates and the flow field around the second phase sphere is assumed to be approximated by the external flow of Hadamard [7] and Rybczynsky [8]. Since only the second phase with uniform constant temperature and concentrations is considered, the internal flow within itself is not considered.

The spherical second phase is characterized by the following parameters: initial radius, R_0, density, ρ_d, viscosity, μ_d, latent heat of phase transition, $L(T_w)$ (< 0 for endothermic; > 0 for exothermic), and first component saturation concentration, $C_{\mathrm{sat}\,1}(T, C_2, C_3, \ldots, C_{N-1})$; the surrounding first phase is characterized by the following parameters: density, ρ, viscosity, μ, specific heat, C_p, effective thermoconductivity, λ, and effective Fick's diffusion coefficients, D_i $(i = 1, 2, \ldots, N-1)$. The first phase is initially at a uniform temperature T_∞ and solute concentrations $C_{\infty i}$ $(i = 1, 2, \ldots, N-1)$, while the second phase is assumed to have a uniform temperature T_w and solute concentrations C_{di} $(i = 1, 2, \ldots, N-1)$ throughout the growth process. Thus, the mass transfer process within the second phase is not considered here.

During the growth process, i.e. $t \geqslant 0$, the system is described by the following equations,

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \alpha \nabla^2 T, \qquad R(t) \leqslant r \leqslant \infty \qquad (1a)$$

$$\frac{\mathrm{D}C_i}{\mathrm{D}t} = \mathrm{D}_i\nabla^2 C_i, \qquad R(t) \leqslant r \leqslant \infty \qquad (1\mathrm{b})$$

with

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + (V_{rt} + V_{rg})\frac{\partial}{\partial r} + \frac{V_{\theta t}}{r}\frac{\partial}{\partial \theta}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$$

$$V_{rg} = \frac{R^2}{r^2}\cdot\left(1 - \frac{\rho_d}{\rho}\right).\dot{R}$$

$$V_{rt} = -U_\infty\cdot\left[1 - \frac{3\kappa + 2}{2(\kappa + 1)}\cdot\frac{R}{r} + \frac{\kappa}{2(\kappa + 1)}\cdot\frac{R^3}{r^3}\right].\cos\theta$$

$$V_{\theta t} = U_\infty\cdot\left[1 - \frac{3\kappa + 2}{4(\kappa + 1)}\cdot\frac{R}{r} - \frac{\kappa}{4(\kappa + 1)}\frac{R^3}{r^3}\right].\sin\theta$$

$$T(r, \theta, 0) = T_\infty \qquad (2\mathrm{a})$$

$$C_i(r, \theta, 0) = C_{\infty i} \qquad (2\mathrm{b})$$

$$T(\infty, \theta, t) = T_\infty \qquad (3\mathrm{a})$$

$$C_i(\infty, \theta, t) = C_{\infty i} \qquad (3\mathrm{b})$$

$$T(R(t), \theta, t) = T_w \qquad (4\mathrm{a})$$

$$C_i(R(t), \theta, t) = C_{wi} \qquad (4\mathrm{b})$$

$$\rho_d\dot{R} = \frac{\lambda}{-L(T_w)}\cdot\frac{1}{2}\int_0^\pi\left(\frac{\partial T}{\partial r}\right)_{r=R(t)}\sin\theta\,\mathrm{d}\theta \qquad (5\mathrm{a})$$

$$\rho_d\dot{R} = \frac{D_i\rho}{C_{di} - C_{wi}}\cdot\frac{1}{2} \times \int_0^\pi\left(\frac{\partial C_i}{\partial r}\right)_{r=R(t)}\sin\theta\,\mathrm{d}\theta \qquad (5\mathrm{b})$$

$$R(0) = R_0 \qquad (6)$$

where i runs from 1 to $N - 1$, $\alpha \equiv \lambda/(\rho C_p)$ is the thermal diffusivity of the surrounding first phase, $\kappa \equiv \mu_d/\mu$ the ratio of the viscosities of the second and first phases, and the first component surface concentration is assumed to be $C_{w1} = C_{\mathrm{sat}1}(T_w, C_{w2}, \ldots, C_{wN-1})$.

The problem is to find the *a priori* unknown interface temperature T_w and concentrations C_{wi} $(i = 1, 2, \ldots, N - 1)$ and obtain the growth law of the second phase, $R(t)$.

METHOD OF SOLUTION

The key to this physically important problem is to recognize that the growth laws obtained from either heat or N-component mass transfer viewpoints must be identical. Thus, one obtains the compatibility conditions from which T_w and C_{wi} $(i = 1, 2, \ldots, N - 1)$ are calculated (see below). The exact solution of this very complicated problem is still yet to be found. However, for certain asymptotic extremes, various kinds of valid approximations are available.

(i) *Boundary layer approximation for the small density ratio ρ_d/ρ case*

With the small density ratio

$$1 \gg \rho_d/\rho \qquad (7)$$

and the thin boundary layer assumptions, i.e.

$$\frac{\partial^2 T}{\partial r^2} \gg \frac{2}{r}\frac{\partial T}{\partial r} \qquad (8\mathrm{a})$$

$$\frac{\partial^2 C_i}{\partial r^2} \gg \frac{2}{r}\frac{\partial C_i}{\partial r} \qquad (8\mathrm{b})$$

$$\frac{\partial^2 T}{\partial r^2} \gg \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) \qquad (8\mathrm{c})$$

$$\frac{\partial^2 C_i}{\partial r^2} \gg \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial C_i}{\partial\theta}\right) \qquad (8\mathrm{d})$$

and

$$0 \leqslant \frac{y}{R} \equiv \frac{r - R(t)}{R(t)} \ll 1 \qquad (8\mathrm{e})$$

the governing equations (1)–(6) are simplified into the following form [9],

$$\frac{\partial T}{\partial t} - y\left(\frac{1}{1 + \kappa}\frac{U_\infty}{R}.\cos\theta + \frac{2}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right)\frac{\partial T}{\partial y}$$

$$+\frac{1}{2(1+\kappa)}\frac{U_\infty}{R}.\sin\theta\frac{\partial T}{\partial\theta}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (9a)$$

$$\frac{\partial C_i}{\partial t}-y\left(\frac{1}{1+\kappa}.\frac{U_\infty}{R}.\cos\theta+\frac{2\,\mathrm{d}R}{R\,\mathrm{d}t}\right)\frac{\partial C_i}{\partial y}$$

$$+\frac{1}{2(1+\kappa)}.\frac{U_\infty}{R}.\sin\theta\frac{\partial C_i}{\partial\theta}=D_i\frac{\partial^2 C_i}{\partial y^2} \quad (9b)$$

$$T(y,\theta,0)=T_\infty \quad (10a)$$

$$C_i(y,\theta,0)=C_{\infty i} \quad (10b)$$

$$T(\infty,\theta,t)=T_\infty \quad (11a)$$

$$C_i(\infty,\theta,t)=C_{\infty i} \quad (11b)$$

$$T(0,\theta,t)=T_w \quad (12a)$$

$$C_i(0,\theta,t)=C_{wi} \quad (12b)$$

$$\rho_d\dot{R}=\frac{\lambda}{-L(T_w)}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial T}{\partial y}\right)_{y=0}\sin\theta\,\mathrm{d}\theta \quad (13a)$$

$$\rho_d\dot{R}=\frac{D_i\rho}{C_{di}-C_{wi}}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial C_i}{\partial y}\right)_{y=0}\sin\theta\,\mathrm{d}\theta \quad (13b)$$

$$R(0)=R_0. \quad (14)$$

From the heat transfer viewpoint, i.e. equations (9a), (10a), (11a), (12a), (13a) and (14), the temperature variable $T(y,\theta,t)$ satisfies the same boundary value problem as in [9]. Thus, one gets [9]

$$R_h(t)=R_0-\frac{Na_h^\theta}{2}\sqrt{\left(\frac{\alpha}{\pi}\right)}\cdot G_h(t) \quad (15)$$

where

$$Na_h^\theta\equiv\frac{\rho}{\rho_d}.B_h^\theta\equiv\frac{\rho}{\rho_d}.\frac{C_p(T_\infty-T_w)}{L(T_w)} \quad (16)$$

and $G_h(t)$ is given by $G(t)$

$$G(t)\equiv\int_0^t\int_0^\pi\frac{\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\tau}{\left(\int_0^\tau\exp\left\{\int_\tau^p\left[6A(\xi).\frac{1-(\tan^2\theta/2).\exp[3\int_\tau^\xi A(s)\,\mathrm{d}s]}{1+(\tan^2\theta/2).\exp[3\int_\tau^\xi A(s)\,\mathrm{d}s]}+4B(\xi)\right]\mathrm{d}\xi\right\}\mathrm{d}p\right)^{\frac{1}{2}}} \quad (17)$$

with

$$A(t)=A_h(t)\equiv\frac{U_\infty(R_h(t))}{3(1+\kappa)R_h(t)} \quad (18)$$

and

$$B(t)=B_h(t)\equiv\frac{\mathrm{d}\ln R_h(t)}{\mathrm{d}t}. \quad (19)$$

From the ith component mass transfer viewpoint, i.e. equations (9b), (10b), (11b), (12b), (13b) and (14), the concentration variable $C_i(y,\theta,t)$ satisfies the same boundary value problem as in [9]. Thus, one gets [9]

$$R_{mi}(t)=R_0-\frac{Na_{mi}^\theta}{2}\sqrt{\left(\frac{D_i}{\pi}\right)}\cdot G_{mi}(t) \quad (20)$$

where

$$Na_{mi}^\theta\equiv\frac{\rho}{\rho_d}.B_{mi}^\theta\equiv\frac{\rho}{\rho_d}.\frac{C_{wi}-C_{\infty i}}{C_{di}-C_{wi}} \quad (21)$$

and $G_{mi}(t)$ is given by $G(t)$ (equation (17)) with

$$A(t)=A_{mi}(t)\equiv\frac{U_\infty(R_{mi}(t))}{3(1+\kappa)R_{mi}(t)} \quad (22)$$

and

$$B(t)=B_{mi}(t)\equiv\frac{\mathrm{d}\ln R_{mi}(t)}{\mathrm{d}t}. \quad (23)$$

The uniqueness of the growth law of the second phase, i.e. $R_h(t)=R_{m1}(t)=R_{m2}(t)=\ldots=R_{mN-1}(t)=R(t)$, gives the following compatibility conditions:

$$\begin{aligned}Na_h^\theta\cdot\sqrt{\alpha}&=Na_{m1}^\theta\cdot\sqrt{D_1}\\&=Na_{m2}^\theta\cdot\sqrt{D_2}\\&=\text{— — —}\\&=Na_{mN-1}^\theta\cdot\sqrt{D_{N-1}}\end{aligned} \quad (24)$$

or

$$\begin{aligned} B^g_h \cdot \sqrt{\alpha} &= B^g_{m1} \cdot \sqrt{D_1} \\ &= B^g_{m2} \cdot \sqrt{D_2} \\ &= - \quad - \quad - \\ &= B^g_{mN-1} \cdot \sqrt{D_{N-1}}. \end{aligned} \tag{25}$$

The values of T_w and C_{wi} $(i = 1, 2, \ldots, N-1)$ must be properly chosen so that the compatibility conditions, equation (25), are satisfied. Then, the required growth law of the second phase is given by

$$R(t) = R_0 - \frac{Na^g_h}{2} \cdot \sqrt{\left(\frac{\alpha}{\pi}\right)} \cdot G(t) \tag{26a}$$

$$= R_0 - \frac{Na^g_{mi}}{2} \cdot \sqrt{\left(\frac{D_i}{\pi}\right)} \cdot G(t) \tag{26b}$$

where $G(t)$ is given by equation (17) with

$$A(t) = \frac{U_\infty(R(t))}{3(1+\kappa)R(t)} \tag{27}$$

and

$$B(t) = \frac{\mathrm{d}\ln R(t)}{\mathrm{d}t}. \tag{28}$$

It is noteworthy that the parameters characterizing the velocity flow field in the surrounding first phase do not appear in the compatibility conditions, equation (25), at all. Equation (25) can be shown to be identical to equation (16) of Part I, if one brings in the same asymptotic approximation in Part I (see [2] for the case of $N = 2$). In other words, the fact that the second phase is moving slowly, does not come into play at all as far as calculating the *a priori* unknown second phase sphere temperature T_w and surface concentrations C_{wi} $(i = 1, 2, \ldots, N-1)$ is concerned. Another noteworthy result is that the spherical second phase initial size R_0 does not appear in the compatibility conditions, equation (25). To fix the ideas, we will consider the following physically important asymptotic cases.

Case 1: $N = 2$ (29)

When $N = 2$, i.e. two-component environment, the main results obtained in [2] are recovered, as expected.

Case 2: $N = 3$ (30)

When $N = 3$, i.e. three-component environment, equation (25) degenerates into the following form

$$\begin{aligned} B^g_h \cdot \sqrt{\alpha} &= B^g_{m1} \cdot \sqrt{D_1} \\ &= B^g_{m2} \cdot \sqrt{D_2}. \end{aligned} \tag{31}$$

Owing to $C_{\mathrm{sat}1}(T, C_2)$ relation, equation (31) determines unique values for T_w, $C_{w1} = C_{\mathrm{sat}1}(T_w, C_{w2})$, and C_{w2}. Then, the required growth law of the second phase is given by either equation (26a) or (26b). Treating the first component as the main solute, the second component as the impurity, and the third component as the solvent, this is the case when the growth of the slowly moving spherical second phase is governed by simultaneous heat and mass transfer limitations in the presence of an impurity.

(ii) *Boundary layer approximation for the large density ratio* ρ_d/ρ *case*

With the large density ratio

$$1 \ll \rho_d/\rho \tag{32}$$

and the thin boundary layer assumptions, i.e.

$$\frac{\partial T}{\partial r^2} \gg \frac{2}{r}\frac{\partial T}{\partial r} \tag{33a}$$

$$\frac{\partial^2 C_i}{\partial r^2} \gg \frac{2}{r}\frac{\partial C_i}{\partial r} \tag{33b}$$

$$\frac{\partial^2 T}{\partial r^2} \gg \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) \tag{33c}$$

$$\frac{\partial^2 C_i}{\partial r^2} \gg \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial C_i}{\partial \theta}\right) \tag{33d}$$

and

$$0 \leqslant \frac{y}{R} \equiv \frac{r = R(t)}{R(t)} \ll 1 \tag{33e}$$

the governing equations (1)–(6) are simplified into the following form [10, 11]

$$\frac{\partial T}{\partial t} - y\,\frac{1}{1+\kappa}\cdot\frac{U_\infty}{R}\cos\theta\,.\frac{\partial T}{\partial y} + \frac{1}{2(1+\kappa)} \times \frac{U_\infty}{R}\,.\sin\theta\,.\frac{\partial T}{\partial\theta} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (34a)$$

$$\frac{\partial C_i}{\partial t} - y\,.\frac{1}{1+\kappa}\cdot\frac{U_\infty}{R}\,.\cos\theta\,.\frac{\partial C_i}{\partial y} + \frac{1}{2(1+\kappa)} \times \frac{U_\infty}{R}\,.\sin\theta\frac{\partial C_i}{\partial\theta} = D_i\frac{\partial^2 C_i}{\partial y^2} \qquad (34b)$$

$$T(y,\theta,0) = T_\infty \qquad (35a)$$

$$C_i(y,\theta,0) = C_{\infty i} \qquad (35b)$$

$$T(\infty,\theta,t) = T_\infty \qquad (36a)$$

$$C_i(\infty,\theta,t) = C_{\infty i} \qquad (36b)$$

$$T(0,\theta,t) = T_w \qquad (37a)$$

$$C_i(0,\theta,t) = C_{wi} \qquad (37b)$$

$$\rho_d\dot{R} = \frac{\lambda}{-L(T_w)}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial T}{\partial y}\right)_{y=0}.\sin\theta\,\mathrm{d}\,\theta \qquad (38a)$$

$$\rho_d\dot{R} = \frac{D_i\rho}{C_{di}-C_{wi}}\cdot\frac{1}{2}\cdot\int_0^\pi\left(\frac{\partial C_i}{\partial y}\right)_{y=0}.\sin\theta\,\mathrm{d}\,\theta \qquad (38b)$$

$$R(0) = R_0. \qquad (39)$$

From the heat transfer viewpoint, i.e. equations (34a), (35a), (36a), (37a), (38a) and (39), the temperature variable $T(y,\theta,t)$ satisfies the same boundary value problem as in [10] and [11]. Thus, one gets [10, 11]

$$R_h(t) = R_0 - \sqrt{\left(\frac{2\alpha}{\pi}\right)}\,.Na_h^g\,.H_h(t) \qquad (40)$$

where $H_h(t)$ is given by $H(t)$

$$H(t) \equiv \int_0^t \frac{\sqrt{\gamma(\tau)}}{\pi} \times \int_0^\pi \frac{\sin^3\theta\,\mathrm{d}\theta\mathrm{d}\tau}{\{[f(\tau,\theta)-\cos\theta]-\frac{1}{3}[f^3(\tau,\theta)-\cos^3\theta]\}^{\frac{1}{2}}} \qquad (41)$$

with

$$f(t,\theta) \equiv \frac{1-(\tan^2\theta/2)\,.\exp[-\gamma(t)\,.t]}{1+(\tan^2\theta/2)\,.\exp[-\gamma(t)\,.t]} \qquad (42)$$

and

$$\gamma(t) = \gamma_h(t) \equiv \frac{U_\infty(R_h(t))}{(1+\kappa)R_h(t)}. \qquad (43)$$

From the ith component mass transfer viewpoint, i.e. equations (34b), (35b), (366), (37b), (38b) and (39), the concentration variable $C_i(y,\theta,t)$ satisfies the same boundary value problem as in [10] and [11]. Thus, one gets [10, 11]

$$R_{mi}(t) = R_0 - \sqrt{\left(\frac{2D_i}{\pi}\right)}.\,Na_{mi}^g\,.\,H_{mi}(t) \qquad (44)$$

where $H_{mi}(t)$ is given by $H(t)$ (equation (41)) with

$$\gamma(t) = \gamma_{mi}(t) \equiv \frac{U_\infty(R_{mi}(t))}{(1+\kappa)R_{mi}(t)}. \qquad (45)$$

The uniqueness of the growth law of the second phase, i.e. $R_h(t) = R_{m1}(t) = R_{m2}(t) = R_{m3}(t) = \ldots = R_{mN-1}(t) = R(t)$, gives the following compatibility conditions

$$\begin{aligned} B_h^g\cdot\sqrt{\alpha} &= B_{m1}^g\cdot\sqrt{D_1} \\ &= B_{m2}^g\cdot\sqrt{D_2} \\ &= \text{—}\quad\text{—}\quad\text{—} \\ &= B_{mN-1}^g\cdot\sqrt{D_{N-1}}. \end{aligned} \qquad (46)$$

The values of T_w and C_{wi} ($i = 1, 2, \ldots, N-1$) must be properly chosen so that the compatibility conditions, equation (46), are satisfied. Then, the required growth law of the second phase is given by

$$R(t) = R_0 - \sqrt{\left(\frac{2\alpha}{\pi}\right)}.\,Na_h^g\,.\,H(t) \qquad (47a)$$

$$= R_0 - \sqrt{\left(\frac{2D_i}{\pi}\right)}.\,Na_{mi}^g\,.\,H(t) \qquad (47b)$$

where $H(t)$ is given by equation (41) with

$$\gamma(t) = \frac{U_\infty(R(t))}{(1+\kappa)R(t)}. \qquad (48)$$

It is noteworthy that both the parameters characterizing the velocity flow field in the surrounding first phase and the spherical second phase initial size R_0 do not appear in the compatibility conditions, equation (46). Equation (46) is identical to equation (25) and can also be shown identical to equation (16) of Part I, if one brings in the same asymptotic approximation, in part I. Thus, within the validity of the approximation, the fact that the second phase is moving slowly, does not come into play at all as far as calculating the *a priori* unknown second phase temperature T_w and surface concentrations C_{wi} $(i = 1, 2, \ldots, N-1)$ is concerned. To fix the ideas, we will consider the following physically important asymptotic cases.

Case 1: $N = 2$ (49)

When $N = 2$, i.e. two-component environment, the main results obtained in [2] are recovered, as expected.

Case 2: $N = 3$ (50)

When $N = 3$, i.e. three-component environment, equation (46) degenerates into the following form

$$\begin{aligned} B_h^g \cdot \sqrt{\alpha} &= B_{m1}^g \cdot \sqrt{D_1} \\ &= B_{m2}^g \cdot \sqrt{D_2}. \end{aligned} \tag{51}$$

Owing to $C_{\text{sat}1}(T, C_2)$ relation, equation (51) determines unique values for T_w, $C_{w1} = C_{\text{sat}1}(T_w, C_{w2})$, and C_{w2}. Then, the required growth law of the second phase is given by either equation (47a) or (47b). Treating the first component as the main solute, the second component as the impurity, and the third component as the solvent, this is the case when the growth of the slowly moving spherical second phase is governed by simultaneous heat and mass transfer limitations in the presence of an impurity.

DISCUSSION

First of all, it is assumed that solute and heat diffusions in the surrounding first phase are adequately described by unsteady state convective diffusion equations with effectively constant Fick's diffusion coefficients and an effectively constant thermoconductivity. It is assumed that all the parameters characterizing second and first phases are effectively constant and there exists a local equilibrium relationship, $C_{\text{sat}1}(T_w, C_{w2}, C_{w3}, \ldots, C_{wN-1})$ at $r = R(t)$ throughout the growth process. The compatibility conditions, equation (25) (for small density ratio ρ_d/ρ) or equation (46) (for large density ratio ρ_d/ρ), are the necessary and sufficient conditions for the existence of the stated constant interface conditions solution, i.e. it guarantees the uniqueness of the growth law of the second phase, $R(t)$. Thus, the basic assumption of strictly constant T_w and C_{wi} $(i = 1, 2, \ldots, N-1)$ is automatically justified *a posteriori* for the second phase problems of the type considered here. Physically, the necessary and sufficient compatibility conditions mean that the second phase can grow if one maintains $T(\infty, t) = T_\infty$ and $C_i(\infty, t) = C_{\infty i}$ $(i = 1, 2, \ldots, N-1)$ throughout the growth process.

CONCLUSIONS

In Part II, two valid approximate treatments of the growth of a slowly moving spherical second phase in the presence of simultaneous heat and N-component mass transfer limitations have been demonstrated. In general, a trial-and-error method must first be used to solve the compatibility conditions, equation (25) or (46), to obtain the *a priori* unknown second phase temperature and surface concentrations. Having thus determined T_w and C_{wi} $(i = 1, 2, \ldots, N-1)$, the growth law of the second phase is then readily obtained. Treating the so-called "impurities" as components in the surrounding first phase, our results should include the growth of a slowly moving spherical second phase as givenned by simultaneous heat and mass transfer limitations in the presence of the impurities as asymptotic cases.

REFERENCES

Given at the end of Part I.

LOI DE CROISSANCE D'UNE SECONDE PHASE SPHERIQUE GOUVERNEE PAR DES CONDITIONS DE TRANSFERTS SIMULTANES DE CHALEUR ET DE MASSE—II

Résumé—On considère dans cette deuxième partie, un nouveau traitement théorique de la croissance d'une seconde phase sphérique en déplacement lent, loi gouvernée par des limitations de transferts simultanés de chaleur et de masse. La nouvelle methode est une extension directe de celle des références (1–3). Il est démontré que la solution de ces cas couplés complexes peut être reliée à des cas non couplés connus. Ainsi, traitant les "impuretés" comme des composants dans la première phase environnante, les résultats peuvent inclure, comme des cas asymptotiques, la croissance d'une seconde phase sphérique en mouvement lent gouvernée par des limitations de transferts simultanés de chaleur et de masse en présence des impuretés.

WACHSTUMSGESETZ EINER KUGELFÖRMIGEN SEKUNDÄRPHASE FÜR GLEICHZEITIGEN WÄRME- UND VIEL-KOMPONENTEN-STOFFÜBERGANG—II

Zusammenfassung—Im Teil II werden neue theoretische Betrachtungen über das Wachstum einer durch gleichzeitigen Wärmeübergang und Mehrkomponentenstoffübergang begrenzten langsam bewegten kugelförmigen Sekundärphase angestellt. Die neue Methode ist eine unmittelbare Erweiterung von [1–3]. Sie zeigt, dass die Lösung dieser komplex überlagerten Fälle auf vorhandene ungekoppelte Fälle übertragen werden kann. Indem man diese sogenannten "Unreinheiten" als Komponenten in der umgebeslen ersten Phase behandelt, sollten unsere Beziehungen das Wachstum einer durch gleichzeitigen Wärme- und Stoffübergang begrenzten langsam bewegte kugelförmigen Sekundärphase in Anwesenheit der Unreinheiten als asymptotische Fälle enthalten.

ЗАКОН РОСТА СФЕРИЧЕСКОЙ ВТОРОЙ ФАЗЫ ПРИ ОДНОВРЕМЕННОМ ПЕРЕНОСЕ ТЕПЛА И МНОГОКОМПОНЕНТОЙ МАССЫ—II

Аннотация—В части II покавана новая теоретическая трактовка роста медленно движущейся сферической второй фазы при совместном переносе тепла и многокомпонентной массью Новый метод является прямым продолжением метода, изложенного в [1–3]. Он показывает, что для решения таких сложных задач взаимосвязанного тепло-и массообмена можно использовать известные для простых задач решения. Таким образом, рассматривая так называемые «примеси» в качестве компонент окружающей их первой фазы, необходимо включать как асимптотический случай рост медленно движущейся сферической второй фазы, определяемый закономерностями одновременного тепло-и массопереноса при наличии примесей.